Mathematics & Statistics

Academic Year: 2017-2018 Marks: 80
Date & Time: 3rd March 2018, 11:00 am
Duration: 3h

Question 1: [12]

Question 1: Select and write the appropriate answer from the given alternative in each of the following sub-question [6]

Question 1.1.1: [2]

If A = $\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is

(A) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

Solution:

If A = $\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is

(A) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

Question 1.1.2: [2]

The principal solutions of sec x = $\frac{2}{\sqrt{3}}$ are ____

$\frac{\pi}{3}, \frac{11\pi}{6}$

$\frac{\pi}{6}, \frac{11\pi}{6}$

$\frac{\pi}{4}, \frac{11\pi}{4}$

$\frac{\pi}{6}, \frac{11\pi}{4}$

Solution:

sec x= $\frac{2}{\sqrt{3}}$

$$\cos x = \frac{\sqrt{3}}{2} = \frac{\cos\pi}{6} = \cos(2\pi - \pi 6)$$

$\frac{\pi}{6}, \frac{11\pi}{6}$

Question 1.1.3: The measure of the acute angle between the lines whose direction ratios are 3, 2, 6 and −2, 1, 2 is ______. **[2]**

Solution:

$$\cos\theta = \left|\frac{3\times -2 + 2\times 1 + 6\times 2}{\sqrt{(3)^2 + (2)^2 + (6)^2}\sqrt{(-2)^2 + 1^2 + 2^2}}\right|$$

$$= \left|\frac{-6+2+12}{\sqrt{49}\sqrt{9}}\right| = \frac{8}{7\times 3} = \frac{8}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{21}\right)$$

Question 1.2: Attempt Any Three of the Following **[8]**

Question 1.2.1: **[4]**

Write the negations of the following statements :
1) All students of this college live in the hostel
2) 6 is an even number or 36 is a perfect square.

Solution: 1) p: All students of this college live in the hostel.

Negation :

~ p: Some students of this college do not live in the hostel.

2) p: 6 is an even number.
q: 36 is a perfect square.

Symbolic form : p v q

∴ ~(p v q) ≡ ~p ∧ ~q

Negation :

6 is not an even number and 36 is not a perfect square.

Question 1.2.2: If a line makes angles α, β, γ with co-ordinate axes, prove that cos 2α + cos2β + cos2γ+ 1 = 0. **[4]**

Solution 1:

Consider cos 2α + cos2β + cos2γ+ 1

$= (2\cos^2\alpha - 1) + (2\cos^2\beta - 1) + (2\cos^2\gamma - 1)$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2(1) - 3 \quad [\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$

$= -1$

$\therefore \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

$\therefore \cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$

Solution 2:

L.H.S: cos 2α + cos2β + cos2γ + 1

$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 + 1$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 2$

= 2 x 1 - 2

= 2- 2

= 0

= R.H.S

Question 1.2.3: Find the distance of the point (1, 2, –1) from the plane x - 2y + 4z - 10 = 0. **[4]**

Solution: The distance of the point $(x_1\ y_1\ z_1)$ to plane ax + by + cz + d = 0

D = $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2+b^2+c^2}}\right|$

∴ $(x_1 y_1 z_1) \equiv (1,2,-1)$

a = 1, b = -2, c = 4

$\therefore D = \left|\frac{1-2(2)+4(-1)-10}{\sqrt{1+4+16}}\right| = \left|\frac{-17}{\sqrt{21}}\right| = \frac{17}{\sqrt{21}}$ units

Question 1.2.4: Find the vector equation of the lines which passes through the point with position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and is in the direction of $-2\hat{i} + \hat{j} + \hat{k}$ [4]

Solution:

Let $\bar{a} = 4\hat{i} - \hat{j} + 2\hat{k}$

$\bar{b} = -\hat{2}i + \hat{j} + \hat{k}$

Equation of the line passing through point $A(\bar{a})$ and having direction $\bar{b}$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$

$\bar{r} = \left(4\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(-2\hat{i} + \hat{j} + \hat{k}\right)$

Question 1.2.5: if $a = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\bar{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{c} = \hat{i} + \hat{j} - \hat{k}$ then find $\bar{a}.\left(\bar{b} \times \bar{c}\right)$ [2]

Solution:

$\bar{a}\left(\bar{b} \times \bar{c}\right) = \left[\bar{a}\bar{b}\bar{c}\right] = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\therefore \bar{a}\left(\bar{b} \times \bar{c}\right) = \begin{vmatrix} 3 & -2 & 7 \\ 5 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$

= 3(-1+2) + 2(-5+2) + 7(5 -1)

= 3-6+28

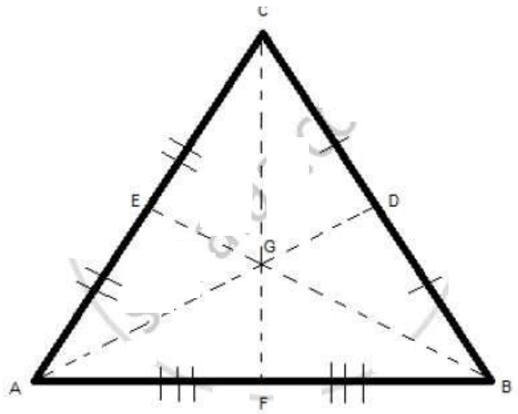
= 25

Question 2: [14]

Question 2: Attempt Any Two of the Following [6]

Question 2.1.1: By vector method prove that the medians of a triangle are concurrent. [3]

Solution:



Let A, B and C be the vertices of a triangle.
Let D, E and F be the midpoints of the sides BC, AC and AB respectively.

Let $\overline{GA} = \bar{a}, \overline{GB} = \bar{b}, \overline{GC} = \bar{c}, \overline{GD} = \bar{d}, \overline{GE} = \bar{e}$ and $\overline{GF} = \bar{f}$ be position vectors of points A, B, C, D, E and F respectively.

Therefore, by midpoint formula,

$$\bar{d} = \frac{\bar{b} + \bar{c}}{2}, \bar{e} = \frac{\bar{a} + \bar{c}}{2} \text{ and } \bar{f} = \frac{\bar{a} + \bar{b}}{2}$$

$$2\bar{d} = \bar{b} + \bar{c}, 2\bar{e} = \bar{a} + \bar{c} \text{ and } 2\bar{f} = \bar{a} + \bar{b}$$

$$2\bar{d} + \bar{a} = \bar{a} + \bar{b} + \bar{c}, 2\bar{e} + \bar{b} = \bar{a} + \bar{b} + \bar{c} \text{ and } 2\bar{f} + \bar{c} = \bar{a} + \bar{b} + \bar{c}$$

$$\frac{2\bar{d} + \bar{a}}{3} = \frac{2\bar{e} + \bar{b}}{3} = \frac{2\bar{f} + \bar{c}}{3} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

$$Let \quad \bar{g} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

$$\therefore \text{ We have } \bar{g} = \frac{\bar{a} + \bar{b} + \bar{c}}{3} = \frac{(2)\bar{d} + (1)\bar{a}}{3} = \frac{(2)\bar{e} + (1)\bar{b}}{3} = \frac{(2)\bar{f} + (1)\bar{c}}{3}$$

If G is the point whose position vector is $\bar{g}$, then from the above equation it is clear that the point G lies on the medians AD, BE, CF
and it divides each of the medians AD, BE, CF internally in the ratio 2:1. Therefore, three medians are concurrent.

Question 2.1.2: Using the truth table, prove the following logical equivalence:
$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ **[3]**

Solution:

1	2	3	4	5	6	7	8
			A			B	
p	q	p ↔ q	p ∧ q	~p	~q	~p ∧ ~q	A V B
T	T	**T**	T	F	F	F	**T**
T	F	**F**	F	F	T	F	**F**
F	T	**F**	F	T	F	F	**F**
F	F	**T**	F	T	T	T	**T**

By column number 3 and 8

p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)

Question 2.1.3: If the origin is the centroid of the triangle whose vertices are A(2, p, −3), B(q, −2, 5) and C(−5, 1, r), then find the values of p, q, r. **[3]**

Solution:

Let $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of $\triangle ABC$ whose vertices are A(2, p, −3), B(q, −2, 5) and C(−5, 1, r)

$\therefore \bar{a} = 2\bar{i} + p\bar{j} - 3\bar{k}, \bar{b} = q\bar{i} - 2\bar{j} + 5\bar{k}, \bar{c} = -5\bar{i} + \bar{j} + r\bar{k}$

Given that origin O is the centroid of $\triangle ABC$

$$\therefore \overline{O} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

$\therefore \bar{a} + \bar{b} + \bar{c} = \overline{O}$

$2\hat{i} + p\hat{j} - 3\hat{k} + \hat{j} - 2\hat{j} + 5\hat{k} - 5\hat{i} + \hat{j} + r\hat{k} = \overline{O}$

$\Rightarrow (2 + q - 5)\hat{i} + (p - 2 + 1)\hat{j} + (-3 + 5 + r)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

by equality of vectors

$2 + q - 5 = 0 \Rightarrow q = 3$

$p - 2 + 1 = 0 \Rightarrow p = 1$

$-3 + 5 + r = 0 \Rightarrow r = -2$

$\therefore$ p = 1, q = 3 and r = -2

Question 2: **[14]**

Question 2.2 | Attempt Any Two of Following **[6]**

Question 2.2.1: Show that every homogeneous equation of degree two in x and y, i.e., $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through origin if $h^2 - ab \geq 0$. **[3]**

Solution 1: Consider a homogeneous equation of the second degree in x and y,

$$ax^2 + 2hxy + by^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Case I: If b = 0 (i.e., a ≠ 0, h ≠ 0), then the equation (1) reduce to $ax^2 + 2hxy = 0$
i.e., x(ax + 2hy) = 0

Case II: If a = 0 and b = 0 (i.e. h ≠ 0), then the equation (1) reduces to 2hxy = 0, i.e., xy = 0 which represents the coordinate axes and they pass through the origin.

Case III: If b ≠ 0, then the equation (1), on dividing it by b, becomes $\frac{a}{b}x^2 + \frac{2hxy}{b} + y^2 = 0$

$$\therefore y^2 + \frac{2h}{b}xy = -\frac{a}{b}x^2$$

On completing the square and adjusting, we get $y^2 + \frac{2h}{b}xy + \frac{h^2x^2}{b^2} = \frac{h^2x^2}{b^2} - \frac{a}{b}x^2$

$$\left(y + \frac{h}{b}x\right)^2 = \left(\frac{h^2 - ab}{b^2}\right)x^2$$

$$\therefore y + \frac{h}{b}x = \pm\frac{\sqrt{h^2 - ab}}{b}x$$

$$\therefore y = -\frac{h}{b}x \pm \frac{\sqrt{h^2 - ab}}{b}x$$

$$\therefore y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b}\right)x$$

∴ equation represents the two lines $y = \left(\frac{-h + \sqrt{h^2 - ab}}{b}\right)x$ and $y = \left(\frac{-h - \sqrt{h^2 - ab}}{b}\right)x$

The above equation are in the form of y = mx

These lines passing through the origin.
Thus the homogeneous equation (1) represents a pair of lines through the origin, if $h^2 - ab \geq 0$.

Solution 2: Consider a homogeneous equation of degree two in x and y

$$ax^2 + 2hxy + by^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

In this equation at least one of the coefficients a, b or h is non zero. We consider two cases.

Case I: If b = 0 then the equation

$$ax^2 + 2hxy = 0$$

$$x(ax + 2hy) = 0$$

This is the joint equation of lines x = 0 and (ax+2hy)=0

These lines pass through the origin.

Case II: If b ≠ 0

Multiplying both the sides of equation (i) by b, we get

$$abx^2 + 2hbxy + b^2y^2 = 0$$

$$2hbxy + b^2y^2 = -abx^2$$

To make LHS a complete square, we add h^2x^2 on both the sides.

$$b^2y^2 + 2hbxy + h^2y^2 = -abx^2 + h^2x^2$$

$$(by + hx)^2 = (h^2 - ab)x^2$$

$$(by + hx)^2 = \left[\left(\sqrt{h^2 - ab}\right)x\right]^2$$

$$(by + hx)^2 - \left[\left(\sqrt{h^2 - ab}\right)x\right]^2 = 0$$

$$\left[(by + hx) + \left[\left(\sqrt{h^2 - ab}\right)x\right]\right]\left[(by + hx) - \left[\left(\sqrt{h^2 - ab}\right)x\right]\right] = 0$$

It is the joint equation of two lines

$$(by + hx) + \left[\left(\sqrt{h^2 - ab}\right)x = 0 \text{ and } (by + hx) - \left[\left(\sqrt{h^2 - ab}\right)x = 0$$

$$\left(h + \sqrt{h^2 - ab}\right)x + by = 0 \text{ and } \left(h - \sqrt{h^2 - ab}\right)x + by = 0$$

These lines pass through the origin when h^2-ab>0

From the above two cases we conclude that the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin.

Question 2.2.2: [4]

In $\triangle ABC$ prove that $\tan\left(\frac{C - A}{2}\right) = \left(\frac{c - a}{c + a}\right)\cot\ \frac{B}{2}$

Solution:

In $\triangle ABC$, by sine Rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=K$$

∴ a = k sinA, b = k sinB, c = k sinC

Consider,

$$\frac{c-a}{c+a}=\frac{k\sin C-k\sin A}{k\sin C+k\sin A}$$

$$=\frac{\sin C-\sin A}{\sin C+\sin A}$$

$$=\frac{2\cos\left(\frac{C+A}{2}\right)\sin\left(\frac{C-A}{2}\right)}{2\sin\left(\frac{C+A}{2}\right)\cos\left(\frac{C-A}{2}\right)}$$

$$=\cot\left(\frac{C+A}{2}\right)\tan\left(\frac{C-A}{2}\right)$$

$$=\tan\frac{B}{2}\tan\left(\frac{C-A}{2}\right)$$

$$\therefore\tan\left(\frac{C-A}{2}\right)=\left(\frac{C-a}{C+a}\right)\cot\frac{B}{2}$$

Hence proved

Question 2.2.3: **[4]**

Find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ using elementary row transformations.

Solution:

$$A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$$

= 1[3] - 2[-1] - 2[2]

$=3+2-4=1\neq 0\Rightarrow A^{-1}$ exist

We know

$$AA^{-1} = I$$

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 + R_1$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 + 2R_3$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - 2R_2 \text{ and } R_3 \to R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & -2 & -4 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$R_1 \to R_1 + 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Question 3: **[14]**

Question 3.1: Attempt Any Two of the Following **[6]**

Question 3.1.1: Find the joint equation of the pair of lines passing through the origin which are perpendicular respectively to the lines represented by $5x^2 + 2xy - 3y^2 = 0$. **[3]**

Solution 1:

Comparing the equation $5x^2 + 2xy - 3y^2 = 0$, we get,

$a = 5, 2h = +2, b = -3$

Let m_1 and m_2 be the slopes of the lines represented by $5x^2 + 2xy - 3y^2 = 0$

$$m_1 + m_2 = -\frac{2h}{b} = -\frac{2}{-3} = \frac{2}{3} \quad \text{......(1)}$$

$$m_1 m_2 = \frac{a}{b} = \frac{5}{-3}$$

Now required lines are perpendicular to these lines

their slopes are $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$

Since these lines are passing through the origin, their separate equations are

$$y = -\frac{1}{m_1}x \text{ and } y = -\frac{1}{m_2}x$$

$$\therefore m_1 y = -x \text{ and } m_2 y = -x$$

$$x + m_1 y = 0 \text{ and } x + m_2 y = 0$$

their combined equation is

$$(x + m_1 y)(x + m_2 y) = 0$$

$$x^2 + (m_1 + m_2)xy + m_1 m_2 y^2 = 0$$

$$x^2 + \frac{2}{3}xy + \frac{-5}{3}y^2 = 0$$

$$3x^2 + 2xy - 5y^2 = 0$$

Solution 2: Given homogeneous equation is

$5x^2 + 2xy - 3y^2 = 0$

Which is factor sable

$5x^2 + 5xy - 3xy - 3y^2 = 0$

$5x(x + y) - 3y(x + y) = 0$

$(x + y)(5x - 3y) = 0$

$\therefore$ $x + y = 0$ and $5x - 3y = 0$ are the two lines represented by the given equation

⇒Their slopes are -1 and 5/3

Required two lines are respectively perpendicular to these lines.

∴ Slopes of required lines are 1 and 3/5 and the lines pass thought origin

∴ Their individual equations are

y = 1.x and $y=-\frac{3}{5}x$

i.e x - y = 0 and 3x + 5y = 0

∴ Their joint equation is

(x - y) (3x + 5y) = 0

$$3x^2-3xy+5xy-5y^2=0$$

$$3x^2+2xy-5y^2=0$$

Question 3.1.2: [3]

Find the angle between the lines $\frac{x-1}{4}=\frac{y-3}{1}=\frac{z}{8}$ and $\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-4}{1}$

Solution:

Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines $\frac{x-1}{4}=\frac{y-3}{1}=\frac{z}{8}$ and $\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-4}{1}$ respectively.

$\therefore \bar{a}=4\hat{i}+\hat{j}+8\hat{k}$ and $\bar{b}=2\hat{i}+2\hat{j}+\hat{k}$

$$\therefore \bar{a}.\bar{b}=4\times2+1\times2+8\times1=8+2+8=18$$

and

$$|\bar{a}|=\sqrt{16+1+64}=\sqrt{81}=9$$

$$\bar{b}=\sqrt{4+1+4}=\sqrt{9}=3$$

Let Θ be the acute angle between the two given lines.

$$\therefore \cos\theta=|\frac{\bar{a}.\bar{b}}{|\bar{a}|.|\bar{b}|}=\frac{18}{9\times3}=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Question 3.1.3: Write converse, inverse and contrapositive of the following conditional statement : If an angle is a right angle then its measure is 90°. [3]

Solution: Converse: If the measure of an angle is 90° then it is a right angle

Inverse : If an angle is not a right angle then its measure is not 90°.

Contrapositive: If the measure of an angle is not 90° then it is not a right angle.

Question 3.2 | Attempt Any Two of the Following [8]

Question 3.2.1: [4]

Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Solution:

Let $\cos^{-1}\frac{12}{13} = x$

$\therefore \cos x = \frac{12}{13}$

$\therefore \sin x = \frac{5}{13}$

and let $\sin^{-1}\frac{3}{5} = y$

sin y = $\frac{3}{5}$

$\therefore \cos y = \frac{4}{5}$

∴ using sin (x + y) = sin x cos y + cos x sin y

$= \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5}$

$= \frac{20 + 36}{13 \times 5}$

= $\frac{56}{65}$

∴ x + y = $\sin^{-1}\frac{56}{65}$

$\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

Hence proved.

Question 3.2.2: Find the vector equation of the plane passing through the points A(1, 0, 1), B(1, −1, 1) and C(4, −3, 2). [4]

Solution: Let the p.v. of points A(1, 0, 1), B(1, −1, 1) and C(4, −3, 2) be

$\vec{a} = \bar{i} + \bar{k}, \bar{b} = \bar{i} - \bar{j} + \bar{k}$ and $\bar{c} = 4\bar{i} - 3\bar{j} + 2\bar{k}$

$\bar{b} - \bar{a} = -\bar{j}, \bar{c} - \bar{a} = 3\bar{i} - 3\bar{j} + \bar{k}$

$$(\bar{b} - \bar{a}) \times (\bar{c} - \bar{a}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -1 & 0 \\ 3 & -3 & 1 \end{vmatrix} = -\bar{i} + 3\bar{k}$$

Equation of plane through A, B, C in vector form is

$(\bar{r} - \bar{a}).\left[(\bar{b} - \bar{a}) \times (\bar{c} - \bar{a})\right] = 0$

$(\bar{r} - \bar{a}).\left(-\bar{i} + 3\bar{k}\right) = 0$

$\bar{r}.\left(-\bar{i} + 3\bar{k}\right) = \left(\bar{i} + \bar{k}\right).\left(-\bar{i} + 3\bar{k}\right) = -1 + 3 = 2$

$\therefore \bar{r}.\left(-\bar{i} + 3\bar{k}\right) = 2$

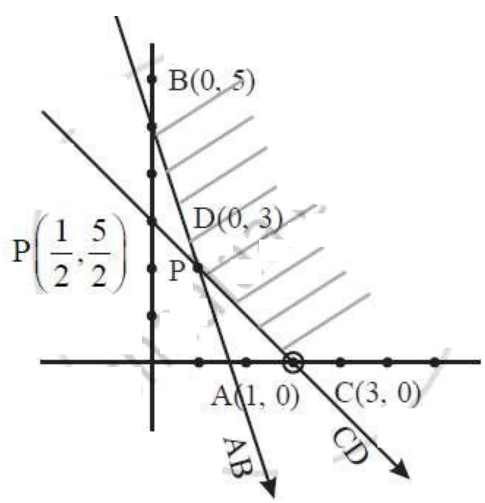
Question 3.2.3: [4]

Solve the following LPP by graphical method:
Minimize Z = 7x + y subject to 5x + y ≥ 5, x + y ≥ 3, x ≥ 0, y ≥ 0

Solution: First we draw the lines AB and CD whose equations are 5x + y = 5 and x + y = 3 respectively.

Line	In equation	Points on x	Points on y	Sign	Feasible region
AB	5x + y = 5	A(1,0)	B(0,5)	≥	Non - origin side AB
CD	x + y = 3	C(3,0)	D(0,3)	≥	Non - origin side of line CD

1 unit = 1 cm both axis



Common feasible region BPC

Points	Minimize z = 7x + y
B(0,5)	Z(B) = 7(0) + 5 = 5
$P\left(\frac{1}{2}, \frac{5}{2}\right)$	$Z(P) = 7 \times \frac{1}{2} + \frac{5}{2} = 6$
C(3,0)	Z(C) = 7x(3) + 0 = 21

Z is minimum at x = 0, y = 5 and min (z) = 5

Question 4: [12]

Question 4.1 | Select and write the appropriate answer from the given alternatives in each of the following sub-questions : [6]

Question 4.1.1: [2]

Let the p. m. f. of a random variable X be __

P(x) = $\frac{3-x}{10}$ for x = -1,0,1,2

= 0 otherwise

Then E(X) is _______.

1

2

0

-1

Solution: 0

x	-1	0	1	2
P(x)	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$
x.P(x)	$\frac{-4}{10}$	0	$\frac{2}{10}$	$\frac{2}{10}$

$$\sum x.P(x) = 0$$

Question 4.1.2: [2]

if $\int_0^k \frac{1}{2+8x^2}dx = \frac{\pi}{16}$ then the value of k is ______.

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

Solution: ½

$$I = \int_0^k \frac{1}{2\left(1+(2x)^2\right)} dx = \frac{\pi}{16}$$

$$\therefore \frac{1}{2} \times \frac{1}{2}\left[\tan^{-1}(2x)\right]_0^k = \frac{\pi}{16}$$

$$\tan^{-1} 2k - \tan^{-1} 0 = \frac{\pi}{4}$$

$$2k = 1$$

$$k = \frac{1}{2}$$

Question 4.1.3: [2]

Integrating factor of linear differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ is _________

$\frac{1}{x^2}$

$\frac{1}{x}$

x

x^2

Solution:

x^2

$$\frac{dy}{dx} + \frac{2y}{x} = x \log x$$

$$P = \frac{2}{x}$$

$$I.F = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$$

Question 4.2: Attempt Any Three of The Following [6]

Question 4.2.1: [2]

Evaluate $\int e^x \left[\frac{\cos x - \sin x}{\sin^2 x}\right] dx$

Solution:

$$I = \int e^x \left[\frac{\cos x}{\sin^2 x} - \frac{\sin x}{\sin^2 x}\right] dx$$

$$= \int e^x \left[\begin{matrix} \cot x . \cos ec x & - \cos ec x \\ f\prime(x) & f(x) \end{matrix}\right]$$

$$\because \int e^x [f(x) + f\prime(x)] dx = e^x f(x) + C$$

$$\therefore I = -e^x . \cos ec x + C$$

Question 4.2.2: [2]

if $y = \tan^2(\log x^3)$, find $\frac{dy}{dx}$

Solution 1:

$$y = [\tan(3 \log x)]^2$$

differentiate w.r.t. x both side

$$\therefore \frac{dy}{dx} = 2[\tan(3 \log x)] \times \sec^2(3 \log x) . \frac{3}{x}$$

$$\therefore \frac{dy}{dx} = \frac{6}{x} \tan(\log x^3) . \sec^2(\log x^3)$$

Solution 2:

Given $y = \tan^2(\log x^3)$

We need to find $\frac{dy}{dx}$

Consider $y = \tan^2(\log x^3)$

$\Rightarrow y = \tan^2(3\log x)$

$\Rightarrow y = [\tan(3\log x)]^2$

Differentiate with respect to xx on both sides we get

$\Rightarrow \frac{dy}{dx} = 2[\tan(3\log x)].\sec^2(3\log x).\frac{3}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{6}{x}.[\tan(3\log x)].\sec^2(3\log x)$

$\therefore \frac{dy}{dx} = \frac{6}{x}.[\tan(\log x^3)].\sec^2(\log x^3)$

Question 4.2.3: **[2]**

Find the area of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$

Solution:

Required area = 4 Area (OAPB)

$= \int_0^1 y dx$

$\therefore \frac{x^2}{1} + \frac{y^2}{4} = 1$

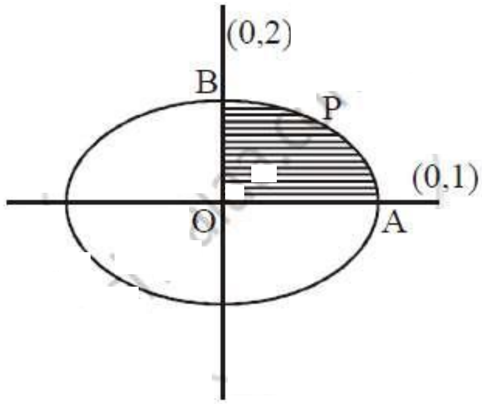
$\therefore y = -2\sqrt{1-x^2}$

$\therefore$Required area = $4\int_0^1 2\sqrt{1-x^2}dx$

$= 8\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right)\right]_0^1$

$= 8\left[\left\{0 + \frac{1}{2}\sin^1(1)\right\} - 0\right]$

$= 8 \times \frac{1}{2}.\frac{\pi}{2} = 2\pi\ sq.units$



Question 4.2.4: Obtain the differential equation by eliminating the arbitrary constants from the following equation: **[2]**

$y = c_1e^{2x} + c_2e^{-2x}$

Solution:

$y = c_1e^{2x} + c_2e^{-2x}$

differentiate w.r.t. x.

$\frac{dy}{dx} = 2c_1e^{2x} - 2c_2e^{-2x}$

Again diff. w.r.t. x.

$\frac{d^2y}{dx^2} = 4c_1e^{2x} + 4c_2e^{-2x}$

$= 4\left(c_1e^{2x} + c_2e^{-2x}\right)$

= 4y

$\therefore \frac{d^2y}{dx^2} - 4y = 0$

Question 4.2.5: **[2]**

Given X ~ B (n, p)
If n = 10 and p = 0.4, find E(X) and var (X).

Solution: Given, n = 10, p = 0.4
q = 1 − p = 1 − 0.4 = 0.6
Now, E(X) = np = 10 x 0.4 = 4
Var(X) = npq = 10 x 0.4 x 0.6 = 2.4

Question 5: [14]

Question 5.1: Attempt any TWO of the following [6]

Question 5.1.1: [3]

Evaluate $\int \frac{1}{3+2\sin x + \cos x} dx$

Solution:

Let I = $\int \frac{1}{3+2\sin x+\cos x} dx$

Put tan $\frac{x}{2} = t$ Then $dx = \frac{2}{1+t^2} dt$

$\sin x = \frac{2t}{1+t^2}$ and $\cos x = \frac{1-t^2}{1+t^2}$

$$\therefore I = \int \frac{2dt/(1+t^2)}{3+2\left(\frac{2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= 2\int \frac{dt/(1+t^2)}{\frac{3(1+t^2)+4t+(1-t^2)}{1+t^2}}$$

$$= 2\int \frac{dt}{2t^2+4t+4} = \int \frac{dt}{(t+1)^2+1}$$

$$= \tan^{-1}(t+1)+c$$

$$= \tan^{-1}\left[\tan\left(\frac{x}{2}\right)+1\right)\Big] + c$$

Question 5.1.2: [3]

If $x = a\cos^3 t,\ y = a\sin^3 t,$

Show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

Solution:

We have $\frac{dy}{dx} = \frac{\mathrm{dy/dx}}{\mathrm{dx/dt}}, \mathrm{dx/dt} \neq 0$ ...(1)

Now $y = a\sin^3 t = a(\sin t)^3 \Rightarrow \sin t = \left(\frac{y}{a}\right)^{\frac{1}{3}}$

$\therefore \frac{dy}{dx} = a\frac{d}{dt}(\sin t)^3 = a.3(\sin t)^2 \frac{d}{dt}(\sin t)$

$= 3a\sin^2 t \cos t$ ... (2)

Also, $x = a\cos^3 t = a(\cos t)^3 \Rightarrow \cos t = \left(\frac{x}{a}\right)^{\frac{1}{3}}$

$\therefore \frac{dx}{dt} = a.3\cos^2 t \frac{d}{dt}(\cos t)$

$= 3a\cos^2 t(-\sin t)$

$= -3a\cos^2 t \sin t$ ...3

From (1), (2) and (3),

$\frac{dy}{dx} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

Question 5.1.3: [3]

Examine the continuity of the function:

f(x) = $\frac{\log 100 + \log(0.01 + x)}{3\mathrm{x}}$, for $x \neq 0 = \frac{100}{3}$ for x = 0; at x = 0.

Solution:

for $x \neq 0, f(x) = \frac{\log 100 + \log(0.01 + x)}{3x} = \frac{\log(1 + 100x)}{3x}$

For continuity at x = 0, $f(0) = f(0^-) = f(0^+)$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\log(1 + 100x)}{3x} = \lim_{x \to 0^-} \frac{1}{(1 + 100x)(-3)}(-100) = \frac{100}{3}$

$\therefore \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\log(1 + 100x)}{3x} = \lim_{x \to 0^+} \frac{1}{(1 + 100x)3}(100) = \frac{100}{3}$

As $f(0) = f(0^-) = f(0^+) = \frac{100}{3}$, the function is continuous at x= 0 .

Question 5.2: Attempt any TWO of the following [8]

Question 5.2.1: Examine the maxima and minima of the function f(x) = $2x^3 - 21x^2 + 36x - 20$. Also, find the maximum and minimum values of f(x). **[4]**

Solution 1:

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$

$$f'(x) = 6x^2 - 42x + 36$$

For finding critical points, we take f'(x)=0

$$\therefore 6x^2 - 42x + 36 = 0$$

$$x^2 - 7x + 6 = 0$$

(x-6)(x-1)=0

For finding the maxima and minima, find f''(x)

f'(x)=12x-42

For x=6

f''(6)=30>0

Minima

For x=1

f''(1)=-30<0

Maxima

Maximum values of f(x) for x=1

f(1)=-3

Minimum values of f(x) for x=6

f(6)=-128

∴ The maximum values of the function is -3 and the minimum value of the function is -128.

Solution 2: f(x) = $2x^3 - 21x^2 + 36x - 20$

$$\therefore f'(x) = 2(3x^2) - 21(2x) + 36(1) - 1$$

$$= 6x^2 - 42x + 36 = 6(x^2 - 7x + 6)$$

= 6(x - 1)(x - 6)

f has a maxima/minima if f'(x) = 0

i.e if 6(x - 1)(x-6) = 0

i.e if x - 1= 0   or x - 6 = 0

i.e if x = 0 or x = 6

Now f"(x) = 6(2x) - 42(1) = 12x - 42

∴ f"(1) = 12(1) - 42 = -30

∴ f"(1) < 0

Hence, f has a maximum at x = 1, by the second derivative test.

Also f"(6) = 12(6)- 42 = 30

∴ f"(6) > 0

Hence, f has a minimum at x = 6, by the second derivative test.

Now, the maximum value of f at 1,

$$f(1) = 2(1^3) - 21(1^2) + 36(1) - 20$$

= 2-21+36-20 = -3

and minimum value of f at x = 6

$$f(6) = 2(6^3) - 21(6^2) + 36(1) - 20$$

= 432 - 756 + 216 - 20 = -128

Question 5.2.2: [4]

Prove that $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2\text{a}} \log\left|\frac{a+x}{a-x}\right| + c$

Solution:

$$\int \frac{1}{a^2 - x^2} dx = \int \frac{1}{(a-x)(a+x)} dx$$

$$= \frac{1}{2a} \int \frac{(a-x)+(a+x)}{(a-x)(a+x)} dx$$

$$= \frac{1}{2a} \int \left(\frac{1}{a+x} + \frac{1}{a-x}\right) dx$$

$$= \frac{1}{2a} \left[\int \frac{1}{a+x} dx + \int \frac{1}{a-x} dx\right]$$

$$= \frac{1}{2a} \left[\log|a+x| + \frac{\log|a-x|}{-1}\right] + C = \frac{1}{2a} [\log|a+x| - \log|a-x|] + C$$

$$= \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$$

Question 5.2.3: [4]

Prove that : $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, if f (x) is an even function.

= 0, if f (x) is an odd function.

Solution:

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$$

$$\int_{a}^{a} f(x)dx = I + \int_{0}^{a} f(x)dx$$

$$Now I = \int_{-a}^{0} f(x)dx$$

Put x=-t

dx = - dt

When x = -a, t = a and when x = 0, t = 0

$$I = \int_{a}^{0} f(-t)(-dt)$$

$$= -\int_{a}^{0} f(-t)dt$$

$$= \int_{0}^{a} f(-t)dt.......... \left[\because \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx\right]$$

$$= \int_{0}^{a} f(-x)dt.......... \left[\because \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(t)dx\right]$$

Equation (i) becomes

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx$$

$$= \int_{0}^{a} [f(-x) + f(x)]dx \ldots\ldots. (ii)$$

case 1: If f(x) is an even function, then f(-x) = f(x).

Thus, equation (ii) becomes

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(x)]dx = 2\int_{0}^{a} f(x)dx$$

Case 2: If f(x) is an odd function, then f(-x) = -f(x).

Thus, equation (ii) becomes

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [-f(x) + f(x)]dx = 0$$

Solution 2: We shall use the following results :

$$\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx \qquad \ldots.(1)$$

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(t)dt \qquad \ldots.(2)$$

If c is between a and b, then

$$\int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx \qquad \ldots.(3)$$

Since 0 lies between -a and a, by (3), we have,

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x) = I_1 + I_2 \qquad \ldots.(\text{Say})$$

In I_1, put $x=-t$. Then $dx=-dt$

When $x=-a, -t=-a$ $\therefore t=a$

When $x=0, -t=0$ $\therefore t=0$

$$\therefore \int_{-a}^{0} f(x)\,dx=\int_{a}^{0} f(-t)(-dt)=\int_{a}^{0} f(-t)\,dt$$

$$=\int_{0}^{a} f(-t)\,dt \qquad \text{....}[By\ (1)]$$

$$=\int_{0}^{a} f(-x)\,dx \qquad \text{....}[By\,(2)]$$

$$\therefore \int_{-a}^{a} f(x)\,dx=\int_{0}^{a} f(-x)\,dx+\int_{0}^{a} f(x)\,dx$$

(i) If f is an even function, then $f(-x)=f(x)$ $\therefore$ in this case,

$$\int_{-a}^{a} f(x)\,dx=\int_{0}^{a} f(x)\,dx+\int_{0}^{a} f(x)\,dx=2\int_{0}^{a} f(x)\,dx$$

(ii) If f is an odd function, then $f(-x)=-f(x)$ $\therefore$ in this case,

$$\int_{-a}^{a} f(x)\,dx=\int_{0}^{a} -f(x)\,dx+\int_{0}^{a} f(x)\,dx=-\int_{0}^{a} f(x)\,dx+\int_{0}^{a} f(x)\,dx=0.$$

Question 5.2.3: **[4]**

Prove that : $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, if f (x) is an even function.

= 0, if f (x) is an odd function.

Solution:

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$$

$$\int_{a}^{a} f(x)dx = I + \int_{0}^{a} f(x)dx$$

$$Now I = \int_{-a}^{0} f(x)dx$$

Put x=-t

dx = - dt

When x = -a, t = a and when x = 0, t = 0

$$I = \int_{a}^{0} f(-t)(-dt)$$

$$= -\int_{a}^{0} f(-t)dt$$

$$= \int_{0}^{a} f(-t)dt.......... \left[\because \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx\right]$$

$$= \int_{0}^{a} f(-x)dt.......... \left[\because \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(t)dx\right]$$

Equation (i) becomes

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx$$

$$= \int_{0}^{a} [f(-x) + f(x)]dx....... (ii)$$

case 1: If f(x) is an even function, then f(-x) = f(x).

Thus, equation (ii) becomes

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(x)]dx = 2\int_{0}^{a} f(x)dx$$

Case 2: If f(x) is an odd function, then f(-x) = -f(x).

Thus, equation (ii) becomes

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [-f(x) + f(x)]dx = 0$$

Solution 2: We shall use the following results :

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \qquad ....(1)$$

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt \qquad ....(2)$$

If c is between a and b, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad ....(3)$$

Since 0 lies between -a and a, by (3), we have,

$$\int_{-a}^a f(x)\,dx = \int_{-a}^0 f(x)\,dx + \int_0^a f(x) = I_1 + I_2 \qquad ....(\text{Say})$$

In I_1, put $x = -t$. Then $dx = -dt$

When $x = -a, -t = -a \qquad \therefore t = a$

When $x = 0, -t = 0 \qquad \therefore t = 0$

$$\therefore \int_{-a}^0 f(x)\,dx = \int_a^0 f(-t)(-dt) = \int_a^0 f(-t)\,dt$$

$$= \int_0^a f(-t)\,dt \qquad ....[By\ (1)]$$

$$= \int_0^a f(-x)\,dx \qquad ....[By(2)]$$

$$\therefore \int_{-a}^a f(x)\,dx = \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx$$

(i) If f is an even function, then $f(-x) = f(x)$ $\therefore$ in this case,

$$\int_{-a}^a f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 2\int_0^a f(x)\,dx$$

(ii) If f is an odd function, then $f(-x) = -f(x)$ $\therefore$ in this case,

$$\int_{-a}^a f(x)\,dx = \int_0^a -f(x)\,dx + \int_0^a f(x)\,dx = -\int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 0.$$

Question 6: **[14]**

Question 6.1 | Attempt any TWO of the following **[6]**

Question 6.1.1: [3]

if $f(x) = \frac{x^2-9}{x-3} + \alpha$ for x> 3

=5, for x = 3

$= 2x^2 + 3x + \beta,$ for x < 3

is continuous at x = 3, find α and β.

Solution: ∵f is continuous at x = 3

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} = f(3)$ ......(1)

Now $\lim_{x \to 3^+} f(x) = \lim_{x \to 0} \left(\frac{x^2-9}{x-3} + \alpha\right) = \lim_{x \to 3} \left[\frac{(x-3)(x+3)}{x-3} + \alpha\right]$

$= \lim_{x \to 0} [(x+3) + \alpha] = (3+3) + \alpha = \alpha + 6$

and

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3} (2x^2 + 3x + \beta) = 2(3)^2 + 3(3) + \beta = 18 + 9 + \beta = 27 + \beta$

Also f(3) = 5 ...(given)

∴ From (1) we get α + 6 = 27 + β = 5`

∴ α + 6 = 5 and 27 + β = 5

∴ α = 5 - 6 = -1 and β = 5 - 27 = - 22

⇒ ∴ α = -1 and β = -22

Question 6.1.2: [3]

Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$

Solution:

Let $y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$

$= \tan^{-1}\left(\frac{5x+1}{1+2-x-6x^2}\right)$

$= \tan^{-1}\left(\frac{5x+1}{1-(3x+2)(2x-11)}\right)$

$= \tan^{-1}\left(\frac{(3x+2)+(2x-1)}{(1-(3x+2))(2x-1)}\right)$

$y = \tan^{-1}(3x+2) + \tan^{-1}(2x-1)$

Differentiate w.r.t. x

$$\therefore \frac{dy}{dx} = \frac{3}{1+(3x+2)^2} + \frac{2}{1+(2x-1)^2}$$

$$= \frac{3}{1+9x^2+12x+4} + \frac{2}{1+4x^2-4x+1}$$

$$= \frac{3}{9x^2+12x+5} + \frac{1}{2x^2-2x+1}$$

Question 6.1.3: **[3]**

A fair coin is tossed 9 times. Find the probability that it shows head exactly 5 times.

Solution:

Let X = no. of heads shows

n = 9 p = $\frac{1}{2}$ q = $\frac{1}{2}$

P(X = x) = ${}^nC_x\, p^n . (q)^{n-x}$ X = 0,1.....n

P(X = 5) = ${}^9C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^4$

$$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \frac{1}{2^9}$$

$$= \frac{3024}{24} \times \frac{1}{2^9}$$

$$= \frac{126}{2^9}$$

$$= \frac{126}{512}$$

= 0.2460

Question 6.2 | Attempt any TWO of the following **[8]**

Question 6.2.1: **[4]**

Verify Rolle's theorem for the following function:
f (x) = x^2 - 4x + 10 on [0, 4]

Solution: Since f (x) is a polynomial,

(i) It is continuous on [0, 4]

(ii) It is differentiable on (0, 4)

(iii) f (0) = 10, f (4) = 16 - 16 + 10 = 10

∴f(0) = f(4) = 10

Thus all the conditions on Rolle's theorem are satisfied

The derivative of f (x) should vanish for at least one point c in (0, 4). To obtain the value of c, we
proceed as follows

$f(x) = x^2 - 4x + 10$

$f'(x) = 2x - 4 = 2(x - 2)$

$\therefore f'(x) = 0 \Rightarrow (x - 2) = 0$

$\therefore x = 2$

$\therefore \exists c = 2$ in (0,4)

We know that $2 \in (0, 4)$

Thus Rolle's theorem is verified.

Question 6.2.2: **[4]**

Find the particular solution of the differential equation:

$$y(1+\log x)\frac{dx}{dy} - x\log x = 0$$

when $y = e^2$ and $x = e$

Solution: Given equation is

$$y(1+\log x)\frac{dx}{dy} - x\log x = 0$$

$$\therefore y(1+\log x)\frac{dx}{dy} = x\log x$$

$$\therefore y(1+\log x)dx = x\log x dy$$

Separating the variables

$$\frac{1}{y}dy = \frac{1+\log x}{x\log x}dx$$

Integrating, we have

$$\int \frac{1}{y}dy = \int \frac{1+\log x}{x\log x}dx$$

$\therefore \log|y| = \log|x \log x| + \log c$

$\therefore \log|y| = \log|cx \log x|$

∴ y = cx log x is the general solution

Given x = e, y = e^2

∴ e^2 = c.e.log e

$\therefore e^2 = c.e$

∴ c = e

∴y = ex.logx

Question 6.2.3: Find the variance and standard deviation of the random variable X whose probability distribution is given below : **[4]**

x	0	1	2	3
P(X = x)	1/8	3/8	3/8	1/8

Solution:

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	1/8	0	0
1	3/8	3/8	3/8
2	3/8	6/8	12/8
3	1/8	3/8	9/8
	Total	12/8	24/8 = 3

$$E(X) = \mu = \sum p_i x_i = \frac{12}{8} = \frac{3}{2}$$

$$Var(X) = \sum_{i=1}^{n} p_i x_i^2 - \mu^2$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= 3 - \frac{9}{4}$$

$= \frac{3}{4}$

$\therefore Var(X) = \sigma^2 = \frac{3}{4}$

Standard deivation of (X) = $\sigma_x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$