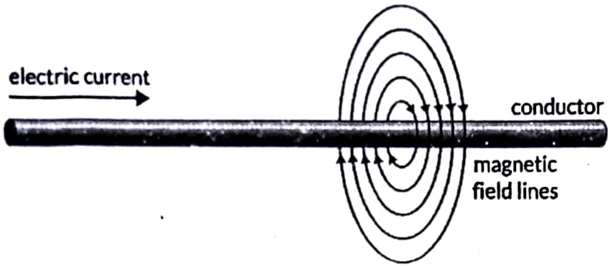


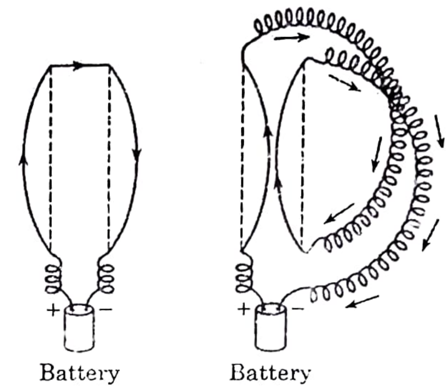
Magnetic Fields Due to Electric Current

SYNOPSIS

Introduction

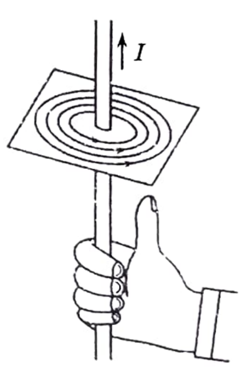
- A current carrying wire produces a magnetic field.
- Oersted suggested an experiment to draw magnetic field lines of magnetic field around the current carrying wire.
- A solenoid is a cylindrical coil of wire acting as a magnet when carrying electric current.
- The magnetic field lines produced by a solenoid is similar to that of a bar magnet, the field lines are closed curves emerging from north pole and merging in the south pole outside the magnet and inside direction is from south to north.
- In both cases, magnetic lines inside the body is strong and uniform, there exists stronger field at the poles compare to middle parts.
- The major difference between magnetic field produced by a bar magnet and the solenoid is magnetism retains in the bar magnet naturally but in solenoid, it is till so long current flows through it.
- The pole of the bar magnet do not lie exactly on the end of the magnet but slightly inside while in solenoid poles can be considered to be lying at the edge.
- Strong magnetic fields are created by high tension power transmission lines, but care has to be taken to reduce the exposure level to less than 0.5 milliguass (mG).

Following figures shows that two wires carrying the current in the same direction attract each other and two wires carrying the current in the opposite direction repel each other as soon as the current starts.



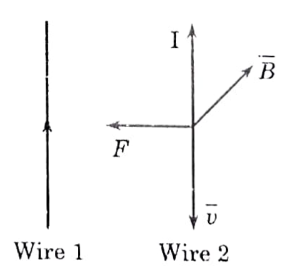
Fleming's Right Hand Rule

It states that, if you hold a straight current carrying conductor in your right hand such that the outstretched thumb gives the direction of current flowing through the conductor, then direction of the curved fingers gives the direction of magnetic field.



Right hand thumb rule

Magnetic Force



Force on wire 2 due to current in wire 1

The magnetic field due to current in wire 1 at any point on wire 2 is directed into plane of paper. Then wire 2 experience a force $\vec{F}$ towards wire 1.

If both electric field $\vec{E}$ and magnetic field $\vec{B}$ are present, the net force on charge q moving with velocity $\vec{v}$ is

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = \vec{F}_e + \vec{F}_m$$

where, $\vec{F}_e \rightarrow$ force due to electric field,

$\vec{F}_m \rightarrow$ force due to magnetic field,

$\vec{F} \rightarrow$ Lorentz force.

(i) If $\vec{v}$ is parallel to $\vec{B}$, $\vec{F}_m = 0$.

(ii) If charge is stationary, *i.e.*, $\vec{v} = 0$, $\vec{F}_m = 0$.

The above figure shows that $\vec{v}$ and $\vec{F}$ are always perpendicular to each other. Hence, $\vec{F} \cdot \vec{v} = 0$ for any magnetic field $\vec{B}$, magnetic force $\vec{F}_m$ is perpendicular to displacement and hence the magnetic force never does any work on moving charges. The magnetic force may change the direction of a charged particle but they can never affect the speed.

$$B = \frac{F_m}{qv}$$

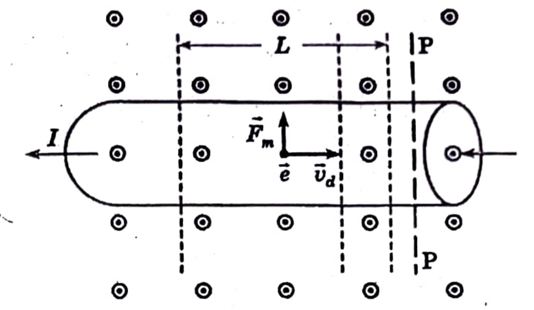
Unit of B is $\frac{\text{N} \cdot \text{s}}{\text{Cm}}$. Its S.I. unit is Tesla (T).

$1\text{ T} = 10^4$ gauss (G)

Dimension of B = $[\text{M}^1 \text{T}^{-2} \text{A}^{-1}]$.

- Magnetic Resonance Imaging (MRI) technique used for medical imaging requires a field strength from 1.5 T to 7 T and nuclear magnetic experiments require a magnetic field upto 14 T.
- Earths magnetic field is about 3.6×10^{-5} T = 0.36 G.

Magnetic Force on a Straight Wire Carrying Current



Electrons in the wire having drift velocity $\vec{v}_d$ experience a magnetic force $\vec{F}_m$ upwards as the applied magnetic field lines come out of the plane of the paper.

Consider a straight wire of length L with magnetic field $\vec{B}$ applied perpendicular to wire coming out of paper, $\vec{v}_d$ is drift velocity then charge q flowing in plane PP will be

$$q = I\,t = \frac{IL}{v_d}$$

$$\vec{F}_m = q(\vec{v}_d \times \vec{B}) = \frac{IL}{v_d}(v_d \cdot B\sin 90°)\,\hat{n}$$

$$= IL \cdot B\sin 90°\,\hat{n}$$

where, $\hat{n}$ is a unit vector $\perp$ to $\vec{B}$ and $\vec{v}_d$.

Therefore, in general

$$\vec{F}_m = I\vec{L} \times \vec{B}$$

Magnetics Force on Arbitrarily Shaped Wire

If I is the current flowing through a segment of infinitesimal length dl along the wire and $\vec{B}$ is perpendicular to magnetic field applied, then

$$d\vec{F}_m = I\,d\vec{l} \times \vec{B}$$

Force on total length L of wire,

$$\vec{F}_m = I\left[\int d\vec{l}\right] \times \vec{B}$$

Magnetic Force on a Closed Circuit

It is given by

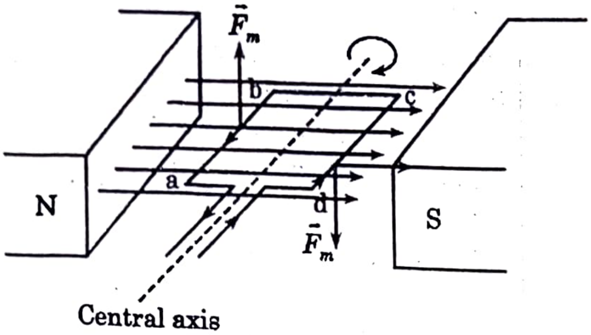
$$\vec{F}_m = \oint_c I\,d\vec{l} \times \vec{B}$$

where, integral is over the closed circuit C.

For uniform $\vec{B}$,

$$\vec{F}_m = 0 \text{ as } \oint_c d\vec{l} = 0.$$

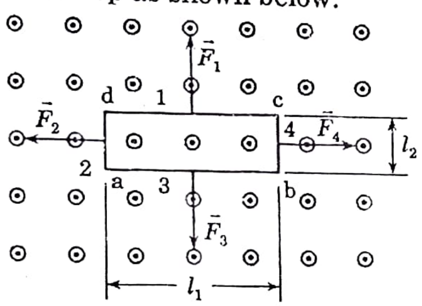
Torque on a Current Loop



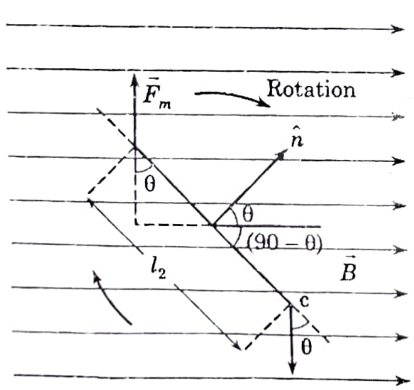
A current loop in a magnetic field : principle of a mtoro

In a current carrying rectangular loop abcd kept in a uniform magnetic field $\vec{F}_m$ acting in opposite directions on segments of loop ab and cd results into the rotation of the loop about its central axis. Here, ab and cd are perpendicular to $\vec{B}$. So we can use Fleming's rule to find out direction of $\vec{F}_m$.

To understand the section of rotation of loop consider the loop as shown below.



Loop abcd placed in a unfirom magnetic field emerging out of the paper. Electric connections are not shown.



Side view of the loop abcd at an angle θ

From figure,

$$\vec{F}_4 = I l_2 B \sin(90 - \theta) = -\vec{F}_2$$

as $\vec{F}_2$ and $\vec{F}_4$ are along the same line and in opposite direction, they cancel each other.

$$\vec{F}_1 = \vec{F}_3 = I l_1 B \sin 90 = I l_1 B$$

as $\vec{F}_1$ and $\vec{F}_3$ are not along the same line, they produce a net torque given by

$$\tau = I l_1 B \frac{1}{2} l_2 \sin\theta + I l_1 B \frac{1}{2} l_2 \sin\theta$$

$$= I l_1 l_2 B \sin\theta$$

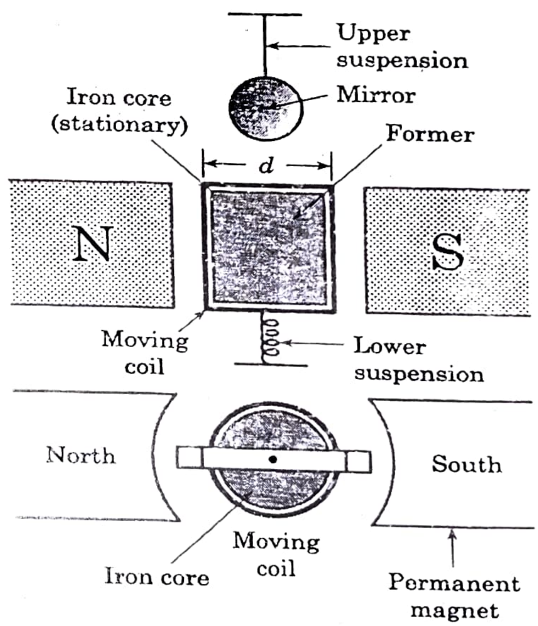
$$= IAB \sin\theta$$

For the coil of N turns,

$$\tau' = NIAB \sin\theta$$

Moving Coil Galvanometer (MCG)

Analog voltmeter and ammeter work on the principle that a current in the circuit or a voltage or a battery can be measured in terms of a torque exerted by a magnetic field on a current carrying coil.



Moving coil galvanometer

In MCG,

Deflecting torque = Restoring torque

$\therefore$ NIBA $= K\phi$

where, K is torsional constant.

$$\therefore \quad \phi = \left(\frac{NBA}{K}\right) I \qquad i.e., \qquad \phi \propto I$$

Magnetic Dipole Moment

Magnetic dipole moment

$\mu = NIA$

When a coil is kept in uniform magnetic field $\vec{B}$,

$$\vec{\tau} = \mu B \sin\theta = \vec{\mu} \times \vec{B}$$

In electric field,

$$\vec{\tau} = \vec{P} \times \vec{E}$$

When $\vec{\mu}$ and $\vec{B}$ are parallel, $\tau = 0$, *i.e.*, minimum.

When $\vec{\mu}$ and $\vec{B}$ are antiparallel, $\tau = \mu B$, *i.e.*, maximum.

Magnetic Potential Energy of a Dipole

In electric field, P.E. of electric dipole is given by

$$U = -\vec{P} \cdot \vec{E}$$

By analogy P.E. of magnetic dipole is given by

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

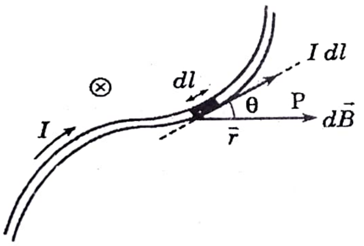
When $\theta = 0$, *i.e.*, $\vec{\mu}$ and $\vec{B}$ are parallel,

$$U_{min} = -\mu B$$

When $\theta = 180$, *i.e.*, $\vec{\mu}$ and $\vec{B}$ are antiparallel,

$$U_{max} = \mu B$$

Biot-Savart Law



A current carrying wire of arbitrary shape, carrying a current *I*. The current in the differential length element *dl* produces differential magnetic field *dB* at a point P at a distance $\vec{r}$ from the element *dl*. The ⊗ indicates that *dB* is directed into the plnae of the paper.

Biot-Savart's law is an equation that gives the magnetic field produced due to a current carrying segment called the current element.

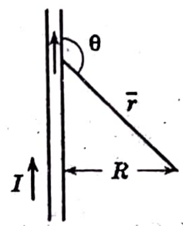
$$|dB| = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

It obeys inverse square law vectorially

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,d\vec{l} \times \vec{r}}{r^3}$$

where direction of magnetic field is given by right hand thumb rule.

Magnetic field due to current in a straight long wire



The magnetic field *dB* at P goving into the plane of the paper, due to current *I* through the wire

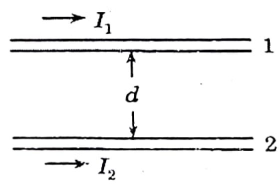
According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

Magnetic field due to semi infinite straight wire is

$$B = \frac{\mu_0 I}{4\pi R}$$

Force of Attraction between the parallel long wires



Two long parallel wires, distance *d* apart

$$B_{21} = \frac{\mu_0 I_1}{2\pi d}$$

Magnetic field at *P* by current lengths elements in upper half of infinitely long wire is equal to magnetic field at *P* by current lengths elements in lower half of infinitely long wire.

$$B = 2\int_0^{\infty} dB = \frac{2\mu_0}{4\pi}\int_0^{\infty} \frac{I\,dl \sin\theta}{r^2}$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi} \cdot R \int_0^{\infty} \frac{dl}{(R^2 + l^2)^{3/2}} \quad \text{as}$$

$$\text{..... } \left[\sin\theta = \frac{R}{r} = \frac{r}{\sqrt{l^2 + R^2}} \right]$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi R} \quad \text{..... [As } l << R]$$

where, B_{21} is magnetic field of second wire due to current I_1 in first wire.

By right hand rule, its direction will be into plane of paper.

According to Lorentz law,

$$F_{21} = I_2\left(\frac{\mu_0 I_1}{2\pi d}\right)\int dl$$

where F_{21} is force on wire 2 due to current I_1 in wire 1.

It is attractive force, *i.e.*, it is towards wire.

By infinitely long wire, force will be infinite

$$\frac{F_{21}}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{F_{12}}{L}$$

where F_{12} is force on wire 1 due to current I_2 in wire 2.

If I_1 and I_2 are antiparallel, force will be repulsive, *i.e.*, $F_{21} = -F_{12}$.

In other words parallel currents attract, antiparallel currents repel.

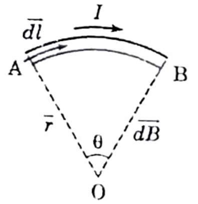
In the formula, $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

if $I_1 = I_2 = 1$ A, $L = 1$ m and $m_0 = 4 \times 10^{-7}$ Wb/Am,

then $\frac{F}{L} = 2 \times 10^{-7}$ N/m.

So, 1 ampere current is the constant current through the wires producing a force of 2×10^{-7} N/m on each other.

Magnetic field produced by a current in a circular arc of a wire



Current carrying wire of a shape of circular arc. The length element $\vec{dl}$ is always perpendicular to $\vec{r}$.

According to Biot Savart's law, magnetic field at centre O is

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I\,d\vec{l} \times \vec{r}}{r^3}$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\;r\sin\theta}{r^3} = \frac{\mu_0 I\,dl}{r^2}$$

The direction of dB is given by right hand rule and it is into the plane of paper.

The total field at O is

$$B = \int dB = \int_{\pi}^{\beta}\frac{\mu_0}{4\pi}\cdot\frac{I\,dl}{r^2}$$

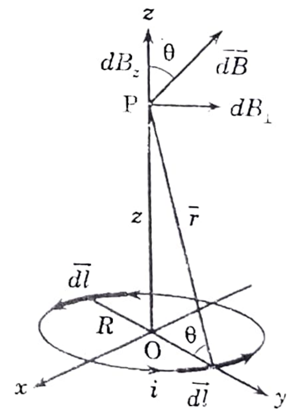
$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{r^2}\int_{\pi}^{\beta} dl = \frac{\mu_0}{4\pi}\cdot\frac{I}{r^2}\int_0^{\theta} r\,d\theta$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\cdot\frac{I\theta}{r}$$

Therefore, magnetic field at the centre of full circle of a wire carrying current I is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot 2\pi = \frac{\mu_0 I}{2\pi r}$$

Axial magnetic field produced by current in a circular loop



Magnetic field on the axis of a circular current loop of radius *R*

Consider a circular loop in xy plane with centre at origin O and carrying current I and having radius R. Then magnetic field at point P on Z axis at a distance r from element $\vec{dl}$ on the loop can be given as

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I(\vec{dl} \times \vec{r})}{r^3}$$

.... [Using Biot-Savart's Law]

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl}{z^2 + R^2}$$

.... [$dl \perp r$ and $r^2 = R^2 + z^2$]

The direction of $d\vec{B}$ is perpendicular to plane termed by $d\vec{l}$ and $\vec{r}$. Its z component is dB_z and the component perpendicular to Z-axis is $dB_\perp$, $\Sigma\, dB_\perp = 0$ as they cancel out due to symmetry.

Net contribution,

$$B_z = \int dB_Z = \int dB\cos\theta$$

$$= \frac{\mu_0}{4\pi}\cdot I\int \frac{dl}{z^2+R^2}\cos\theta$$

$$= \frac{\mu_0}{4\pi}\cdot I\int \frac{R\,dl}{(z^2+R^2)^{3/2}}$$

$$\text{....}\left[\text{As } \cos\theta = \frac{R}{(z^2+R^2)^{1/2}}\right]$$

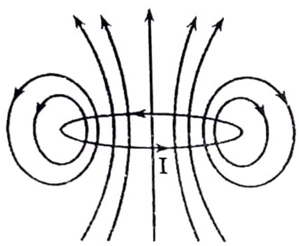
$$= \frac{\mu_0}{4\pi}\cdot\frac{IR}{(z^2+R^2)^{3/2}}\cdot 2\pi R$$

$$\therefore\quad B_z = \frac{\mu_0}{2}\cdot\frac{IR^2}{(z^2+R^2)^{3/2}}$$

Magnetic Lines for a Current Loop

Magnetic field at the centre of loop for the coil of N turns can be obtained by putting $z = 0$ in the above equation

$$B = \frac{\mu_0 NI}{2R}$$



Magnetic field lines for a current loop

The direction of field is as per right hand rule. The upper part of the loop is regarded as North pole and lower part as south pole of a bar magnet.

If $z >> R$,

$$B_z = \frac{\mu_0 IR^2}{2z^3} = \frac{\mu_0 IA}{2\pi z^3} \quad \text{.... } [A = \pi R^2]$$

$$\therefore\quad B_z = \frac{\mu_0}{2\pi}\cdot\frac{m}{z^3}$$

.... [$m = IA$ magnetic moments]

$$\vec{B}_z = \frac{\mu_0}{4\pi}\cdot\frac{2\vec{m}}{z^3}$$

It is analogous to

$$\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 z^3}$$

.... [electric field at an axial point of electric dipole]

Similarly, we can find magnetic field at a distance x on the perpendicular bisector as

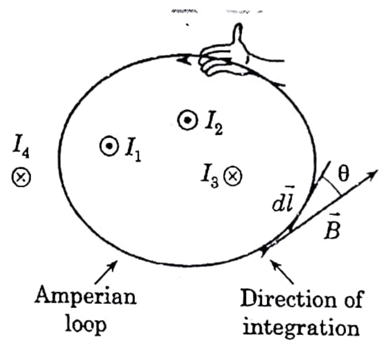
$$\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{\vec{m}}{x^3}, \qquad x >> R.$$

Ampere's Law

- Ampere's law can be derived from Biot and Savart's Law.
- According to Ampere's law,

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

where, $\oint$ indicates that integration is taken over a closed loop called Amperian loop and current I is the net current encircled by Amperian loop.



Amperian loop

- As current goes perpendicular to plane of paper, $\vec{B}$ is in the plane of paper, ⊙ I_1 and I_2 are currents coming out of paper and ⊗ I_3 is the current entering the paper and $d\vec{l}$ length element on Amperian loop.

$$\oint \vec{B}\cdot d\vec{l} = \oint B\cdot dl\cos\theta = \mu_0\,(I_1 + I_2 - I_3)$$

As I_1 and I_3 are parallel to associated thumb, they are positive. So I_3 is negative.

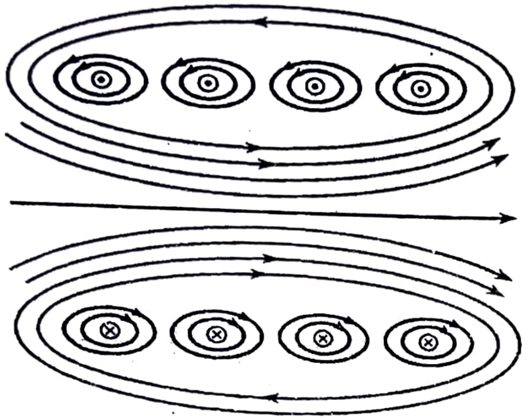
As I_4 is not within Amperian loop, its contribution to B cancel out.

- For a long straight wire carrying a current I, $\vec{B}$ and $d\vec{l}$ are tangential to the Amperian loop (circle)

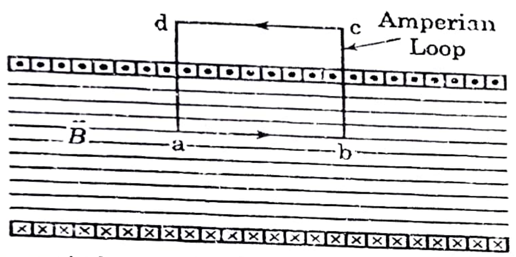
$$\therefore\quad \oint \vec{B}\cdot d\vec{l} = \int_0^{2\pi}\frac{\mu_0 I}{2\pi r}\cdot r\,d\theta = \mu_0 I$$

- Biot-Savart's law in magnetostatics is analogous to Coulomb's law in electrostatics.
- Ampere's law in magnetostatics is analogous to Gauss' law in electrostatics.

Magnetic Field of a Solenoid



Schematic diagram of a cross section of a current carrying solenoid



Ampere's law applied to a part of a long ideal solenoid : The dots ⊙ shows that the current is coming out of the plnae of the paper and the crosses ⊗ shows that the current is goiving into the plane of the paper, both in the coil of square cross section wiere.

For a real solenoid of finite length, magnetic field is uniform having good strength at the centre and weak at outside the coil.

For ideal solenoid length is infinite and wire has a square cross-section and its wound very closely with insulating material, so magnetic field inside the coil is uniform and along the axis of solenoid and outside the solenoid is zero.

For rectangular loop abcd,

$$\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

$$= \mu_0 I$$

$$\therefore \quad B \cdot L + 0 + 0 + 0 = \mu_0 I$$

$$\int_b^c \vec{B} \cdot d\vec{l} = 0 \text{ and } \int_d^a \vec{B} \cdot d\vec{l} = 0$$

As $\vec{B}$ and $d\vec{l}$ are perpendicular

$$\int_c^d \vec{B} \cdot d\vec{l} = 0$$

as wire is outside the solenoid.

If n is no. of turns per unit length of solenoid and i is current flowing through wire, then current coming out of paper,

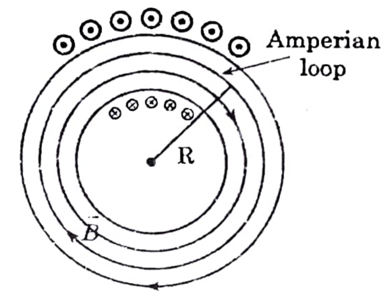
$$I = nLi$$

$$\therefore \quad BL = \mu_0 nLi$$

$$\therefore \quad B = \mu_0 ni$$

Magnetic Field of a Toroid

A toroid is a solenoid of finite length bent into a hollow circular tube like structure similar to a pressurized rubber tube inside a tyre of vehicle.



Amperian loop along the central axis of the toroid

According to Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\therefore \quad B \cdot 2\pi R = \mu_0 NI$$

where, N is no. of turns in a toroid.

$$\therefore \quad B = \frac{\mu_0 NI}{2\pi R}$$

Unlike solenoid, magnetic field is not uniform over the cross section of toroid.

$B = 0$ for points outside ideal toroid.

LIST OF IMPORTANT FORMULAE

1. Magnetic force acting on a charge q moving with velocity v in a magnetic field B is

$$f = qvB \sin\theta$$

2. Lorentz force acting on a charge

$$f = qE + qvB \sin\theta$$

3. Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

4. Force on a straight conductor carrying current I is

$$F = IlB \sin\theta$$

5. **Magnetic induction at a point near an infinitely long straight conductor carrying current I is**

$$B = \frac{\mu_0 I}{2\pi r}$$

6. **Magnetic field at one end point of infinitely long straight wire or semi-infinite straight wire is**

$$B = \frac{\mu_0 I}{4\pi r}$$

7. **Magnetic field at the centre of circular coil of n turns is**

$$B = \frac{\mu_0 n I}{2\pi r}$$

8. **Magnetic field produced by a current in circular arc of a wire is**

$$B = \frac{\mu_0 I \theta}{4\pi r}$$

9. Magnetic field at an axial point due to circular coil is

$$B = \frac{\mu_0}{2} \cdot \frac{I R^2}{(z^2 + R^2)^{3/2}}$$

10. Magnetic force per unit length between two parallel long wires is

$$\frac{F}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{d}$$

11. For cyclotron

(i) Momentum, $mv = qBR$

(ii) Radius, $R = \frac{mv}{Bq}$

(iii) Time period for semicircular motion

$$= \frac{\pi R}{v} = \frac{\pi m}{Bq}$$

(iv) Period of revolution of a charged particle is

$$T = \frac{2\pi m}{Bq}$$

(v) Frequency of revolution is

$$f = \frac{1}{T} = \frac{Bq}{2\pi m}$$

(vi) Maximum speed acquired by ion to emerge from a cyclotron is

$$v_{max} = \frac{BqR}{m}$$

(vii) $KE_{max} = \frac{1}{2} m v_{max}^2 = \frac{B^2 q^2 R^2}{2m}$ (in Joule)

$= \frac{B^2 q^2 R^2}{2me}$ (in eV)

12. **Magnetic moment of a current carrying wire is**

$$M = IA$$

13. **Magnetic moment of a circular coil of n turns is**

$$M = nIA$$

14. **Torque acting on current carrying coil placed in magnetic field is**

$$\tau = NIBA \sin\theta = MB \sin\theta$$

15. **For moving coil galvanometer (M.C.G.)**

(i) Deflecting torque, $\tau_d = NIBA \cos\theta$

(ii) Restoring torque, $\tau_R = k\theta$

(iii) Current, $I = \frac{k\theta}{NBA}$

(iv) Current sensitivity, $S_i = \frac{d\theta}{dI} = \frac{NBA}{k}$

(v) Voltage sensitivity, $S_v = \frac{d\theta}{dv} = \frac{S_i}{G}$

16. Biot-Savart's Law,

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{r^2} \sin\theta$$

17. Magnetic induction due to a long solenoid

(i) at a point inside the solenoid is

$$B_{inside} = \mu_0 n I$$

(ii) at a point near end of solenoid is

$$B_{end} = \frac{\mu_0 n I}{2}$$

where, $n = \frac{N}{L}$ = turns per unit length.

(iii) at a point outside the solenoid,

$$B_{outside} = 0.$$

18. Magnetic induction due to toroid is

$$B_{inside} = \mu_0 n I$$

where, $n = \frac{N}{2\pi r}$ = turns per unit length.

$$B_{outside} = 0.$$

19. Magnetic potential energy of a dipole in a magnetic field is

$$u = -MB\cos\theta$$

20. Magnetic field at a point on the perpendicular bisector is

$$B = \frac{\mu_0}{2\pi} \cdot \frac{m}{x^3}.$$