Chapter 3: Kinetic Theory of Gases and Radiation

EXERCISES [PAGES 73 - 74]

Exercises | Q 1.1 | Page 73

Choose the correct option:
In an ideal gas, the molecules possess

1. **only kinetic energy**
2. both kinetic energy and potential energy
3. only potential energy
4. neither kinetic energy nor potential energy

SOLUTION

Only kinetic energy

Exercises | Q 1.2 | Page 73

Choose the correct option.
The mean free path λ of molecules is given by where n is the number of molecules per unit volume and d is the diameter of the molecules.

$$\sqrt{\frac{2}{\pi n d^2}}$$

$$\frac{1}{\pi n d^2}$$

$$\frac{1}{\sqrt{2}\pi n d^2}$$

$$\frac{1}{\sqrt{2\pi n d^2}}$$

Exercises | Q 1.3 | Page 73

Choose the correct option.
If the pressure of an ideal gas decreases by 10% isothermally, then its volume will _______.

1. decrease by 9%
2. increase by 9%
3. decrease by 10%
4. **increase by 11.11%**

SOLUTION

increase by 11.11%

Explanation:

[Use the formula $P_1V_1 = P_2V_2$ ......(given)

$$\frac{V_2}{V_1} = \frac{1}{0.9} = 1.111$$

$$\therefore \frac{V_2 - V_1}{V_1} = 0.1111,$$

i.e. 11.11%

Exercises | Q 1.4 | Page 73

Choose the correct option.

If a = 0.72 and r = 0.24, then the value of t_r is

1. 0.02
2. **0.04**
3. 0.4
4. 0.2

SOLUTION

0.04

Exercises | Q 1.5 | Page 73

Choose the correct option.

The ratio of emissive power of perfect blackbody at 1327°C and 527°C is

1. 4:1
2. **16:1**
3. 2:1
4. 8:1

SOLUTION

16:1

Exercises | Q 2.1 | Page 73

Answer in brief:

What will happen to the mean square speed of the molecules of a gas if the temperature of the gas increases?

SOLUTION

If the temperature of a gas increases, the mean square speed of the molecules of the gas will increase in the same proportion.

Exercises | Q 2.2 | Page 73

Answer in brief:
On what factors do the degrees of freedom depend?

SOLUTION

The degrees of freedom depends upon
(i) the number of atoms forming a molecule
(ii) the structure of the molecule
(iii) the temperature of the gas.

Exercises | Q 2.3 | Page 73

Answer in brief:
Write ideal gas equation for a mass of 7 g of nitrogen gas.

SOLUTION

Ideal has equation = PV = nRT

Here, n = $\frac{\text{mass of the gas}}{\text{molar mass}} = \frac{7}{28} = \frac{1}{4}$

Therefore, the corresponding ideal gas equation is

PV = $\frac{1}{4}$RT

Exercises | Q 2.4 | Page 73

If the density of oxygen is 1.44 kg/m^3 at a pressure of 10^5 N/m^2, find the root mean square velocity of oxygen molecules.

SOLUTION

Data: ρ = 1.44 kg/m^3, P = 10^5 N/m^2

∴ The root mean square velocity of oxygen molecules,

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.44}} \text{ m/s}$$

$$= \sqrt{2.083 \times 10^5} = \sqrt{20.83 \times 10^4}$$

$= 4.564 \times 10^2$ m/s

Exercises | Q 2.5 | Page 73

Answer in brief:
Define athermanous substances and diathermanous substances.

SOLUTION

Athermanous substances-

Substances that don't allow transmission of infrared radiation through them are called athermanous substances.

- For example, wood, metal, CO2, water vapors, etc.

Diathermanous substances -

Substances that allow transmission of infrared radiation through them are called diathermanous substances.

- For example, rock salt, pure air, glass, etc.

Exercises | Q 3 | Page 73

When a gas is heated, its temperature increases. Explain this phenomenon on the basis of the kinetic theory of gases.

SOLUTION

Molecules of a gas are in a state of continuous random motion. They possess kinetic energy. When a gas is heated, there is an increase in the average kinetic energy per molecule of the gas. Hence, its temperature increases (the average kinetic energy per molecule being proportional to the absolute temperature of the gas).

Exercises | Q 4 | Page 73

Explain, on the basis of the kinetic theory of gases, how the pressure of a gas changes if its volume is reduced at a constant temperature.

SOLUTION

The average kinetic energy per molecule of a gas is constant at a constant temperature. When the volume of a gas is reduced at a constant temperature, the number of collisions of gas molecules per unit time with the walls of the container increases. This increases the momentum transferred per unit time per unit area, i.e., the force exerted by the gas on the walls. Hence, the pressure of the gas increases.

Exercises | Q 5 | Page 73

Answer in brief:
Mention the conditions under which a real gas obeys ideal gas equation.

SOLUTION

A real gas obeys the ideal gas equation when the temperature is very high and pressure is very low
[**Note:** Under these conditions, the density of a gas is very low. Hence, the molecules, on average, are far away from each other. The intermolecular forces are then not of much consequence.]

Exercises | Q 6 | Page 73

Answer in brief:
State the law of equipartition of energy and hence calculate the molar specific heat of monatomic and diatomic gases at constant volume and constant pressure.

SOLUTION

Law of equipartition of energy states that for a dynamical system in thermal equilibrium the total energy of the system is shared equally by all the degrees of freedom. The energy associated with each degree of freedom per molecule is 1/2 k_BT, where k_B is the Boltzmann's constant.

For example, for a monoatomic molecule, each molecule has 3 degrees of freedom. According to kinetic theory of gases, the mean kinetic energy of a molecule is 3/2 k_BT.

Specific heat capacity of Monatomic gas:

The molecules of a monatomic gas have 3 degrees of freedom.

The average energy of a molecule at temperature T is $\frac{3}{2}K_BT$

The total internal energy of a mole is: $\frac{3}{2}N_AK_BT$, where N_A is the Avogadro number.

The molar specific heat at constant volume C_V is

For an ideal gas,

C_v (monatomic gas) = $\frac{dE}{dT} = \frac{3}{2}RT$

For an ideal gas, C_p - C_v = R

where C_p is molar specific heat at constant pressure.

Thus, C_p = $\frac{5}{2}R$

Specific heat capacity of Diatomic gas:

The molecules of a monatomic gas have 5 degrees of freedom, 3 translational, and 2 rotational.

The average energy of a molecule at temperature T is $\frac{5}{2}K_BT$

The total internal energy of a mole is: $\frac{5}{2}N_AK_BT$

The molar specific heat at constant volume C_v is

For an ideal gas,

C_v (monatomic gas) = $\frac{dE}{dT} = \frac{5}{2}RT$

For an ideal gas, C_p - C_v = R

where C_p is the molar specific heat at constant pressure.

Thus, C_p = $\frac{7}{2}R$

A soft or non-rigid diatomic molecule has, in addition, one frequency of vibration which contributes two quadratic terms to the energy. Hence, the energy per molecule of a soft diatomic molecule is

$$E = \left(\frac{1}{2}k_BT\right) + 2\left(\frac{1}{2}k_BT\right) + 2\left(\frac{1}{2}k_BT\right) = \frac{7}{2}k_BT$$

Therefore, the energy per mole of a soft diatomic molecule is

$$E = \frac{7}{2}k_BT \times N_A = \frac{7}{2}RT$$

In this case, $C_v = \frac{dE}{dT} = \frac{7}{2}R$ and

$C_p = C_v + R = \frac{7}{2}R + R = \frac{9}{2}R$

[**Note:** For a monatomic gas, adiabatic constant,

$\frac{C_p}{C_v} = \frac{5}{3}$ for a diatomic gas, $\frac{7}{5}$ or $\frac{9}{7}$}

Exercises | Q 7 | Page 73

Answer in brief:

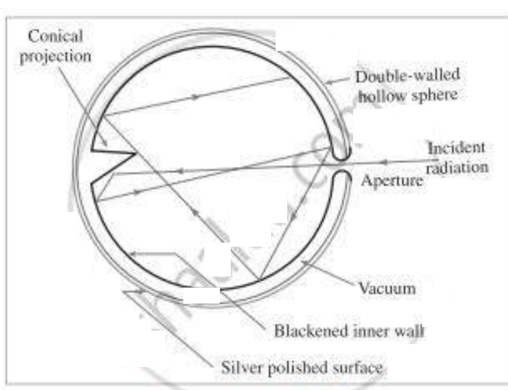
What is a perfect blackbody? How can it be realized in practice?

SOLUTION

A perfect blackbody or simply a blackbody is defined as a body which absorbs all the radiant energy incident on it.

Fery designed a spherical blackbody which consists of a hollow double-walled, metallic sphere provided with a tiny hole or aperture on one side, Fig. The inside wall of the sphere is blackened with lampblack while the outside is silver-plated. The space between the two walls is evacuated to minimize heat loss by conduction and convection.

Any radiation entering the sphere through the aperture suffers multiple reflections where about 97% of it is absorbed in each incidence by the coating of lampblack. The radiation is almost completely absorbed after a number of internal reflections. A conical projection on the inside wall opposite the hole minimizes the probability of incident radiation escaping out.



Fery's blackbody

When the sphere is placed in a bath of suitable fused salts, so as to maintain it at the desired temperature, the hole serves as a source of black-body radiation. The intensity and the nature of the radiation depend only on the temperature of the walls.

A blackbody, by definition, has a coefficient of absorption equal to 1. Hence, its coefficient of reflection and coefficient of transmission are both zero. The radiation from a blackbody, called blackbody radiation, covers the entire range of the electromagnetic spectrum. Hence, a blackbody is called a full radiator.

Exercises | Q 8.1 | Page 73

Answer in brief:
State the Stefan-Boltzmann law.

SOLUTION

The Stefan Boltzmann law describes the power radiated from a black body in terms of its temperature. Specifically, the Stefan-Boltzmann law states that the total energy radiated per unit surface area of a black body across all wavelengths per unit time, is directly proportional to the fourth power of the black body's thermodynamic temperature T:

$j = \sigma T^4$

where $\sigma = 5.670373 \times 10^{-8}$ Wm^{-2} K^{-4}, Stefan Boltzmann constant.

Exercises | Q 8.2 | Page 73

Answer in brief:
State the Wien's displacement law

SOLUTION

According to Wein's displacement law, as the temperature of a black body rises, the peak of the distribution curve shifts towards the shorter wavelength (λ).

Thus, $\lambda_m T = b$, a (constant)
where $b = 2.898 \times 10^{-3}$mK.

Exercises | Q 9 | Page 73

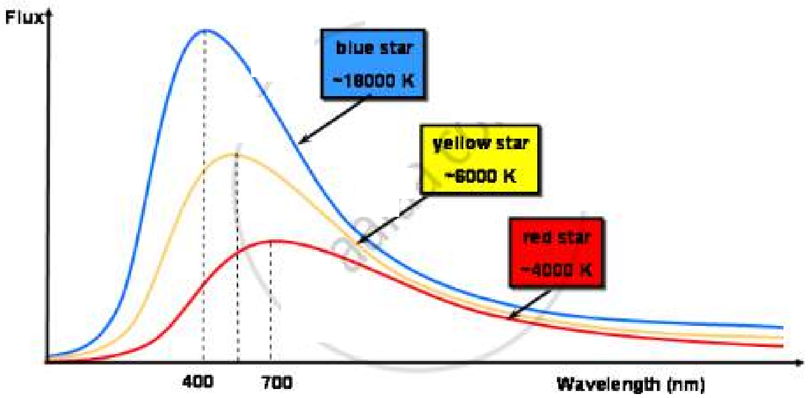
Answer in brief:
Explain the spectral distribution of blackbody radiation.

SOLUTION

All objects with a temperature above absolute zero (0 K, -273.15°C) emit energy in the form of electromagnetic radiation.

A blackbody is a theoretical or model body that absorbs all radiation falling on it, reflecting or transmitting none. It is a hypothetical object which is a “perfect” absorber and a “perfect” emitter of radiation over all wavelengths.

The spectral distribution of the thermal energy radiated by a blackbody (i.e. the pattern of the intensity of the radiation over a range of wavelengths or frequencies) depends only on its temperature.



The characteristics of blackbody radiation can be described in terms of several laws:

1. **Planck’s Law** of blackbody radiation, a formula to determine the spectral energy density of the emission at each wavelength (E_λ) at a particular absolute temperature (T).

$$E_\lambda = \frac{8\pi h\ c}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

2. **Wien’s Displacement Law,** which states that the frequency of the peak of the emission (f_{max}) increases linearly with absolute temperature (T). Conversely, as the temperature of the body increases, the wavelength at the emission peak decreases.
   $f_{max} \propto T$
3. **Stefan–Boltzmann Law**, which relates the total energy emitted (E) to the absolute temperature (T).
   $E \propto T^4$

In the image above, notice that:

- The blackbody radiation curves have quite a complex shape (described by Planck’s Law).
- The spectral profile (or curve) at a specific temperature corresponds to a specific peak wavelength, and vice versa.
- As the temperature of the blackbody increases, the peak wavelength decreases (Wien’s Law).

- The intensity (or flux) at all wavelengths increases as the temperature of the blackbody increases.

- The total energy being radiated (the area under the curve) increases rapidly as the temperature increases (Stefan–Boltzmann Law).

- Although the intensity may be very low at very short or long wavelengths, at any temperature above absolute zero energy is theoretically emitted at all wavelengths (the blackbody radiation curves never reach zero).

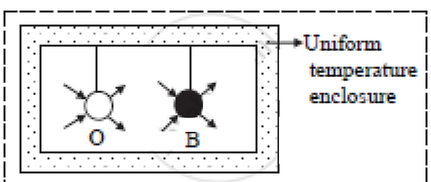
Exercises | Q 10 | Page 73

Prove Kirchhoff's law of radiation theoretically.

SOLUTION

Consider an ordinary body O and perfectly black body B of the same dimension suspended in a uniform temperature enclosure as shown in the figure.
At thermal equilibrium, both the bodies will have the same temperature as that of the enclosure.

Let, E = emissive power of ordinary body O
Eb = emissive power of perfectly black body B
a = coefficient of absorption of O
e = emissivity of O
Q = radiant energy incident per unit time per unit area on each body



Quantity of heat absorbed per unit area per unit time by body O = aQ.
Quantity of heat energy emitted per unit area per unit time by body O = E.
Since there is no change in temperature
E = aQ

Q = E/a ....(1)

Quantity of heat absorbed per unit area per unit time by a perfectly black body, B = Q
The radiant heat energy emitted per unit time per unit area by a perfectly black body, B = Eb
Since there is no change in temperature.
Eb = Q ....(2)

From equations (1) and (2),

$$\frac{E}{a} = E_b \Rightarrow \frac{E}{E_b} = a$$

But, $\frac{E}{E_b} = e$

a = e

Exercises | Q 11 | Page 73

Calculate the ratio of the mean square speeds of molecules of a gas at 30 K and 120 K.

SOLUTION

Data: T_1 = 30 K, T_2 = 120 K

The mean square speed, $\bar{v}^2 = \frac{3RT}{M_0}$

$\therefore \frac{\bar{v}_1^2}{\bar{v}_2^2} = \frac{T_1}{T_2}$ for a given gas

$\therefore \frac{\bar{v}_1^2}{\bar{v}_2^2} = \frac{30K}{120k} = \frac{1}{4}$

This is the required ratio.

Exercises | Q 12 | Page 73

Two vessels A and B are filled with the same gas where the volume, temperature, and pressure in vessel A is twice the volume, temperature, and pressure in vessel B. Calculate the ratio of the number of molecules of the gas in vessel A to that in vessel B.

SOLUTION

Data: $V_A = 2V_B$, $T_A = 2T_B$, $P_A = 2P_B$ $PV = Nk_BT$

$\therefore$ The number of molecules, N = $\frac{PV}{k_BT}$

$\therefore N_A = \frac{P_AV_A}{k_BT_A}$ and $N_B = \frac{P_BV_B}{k_BT_B}$

$$\therefore \frac{N_A}{N_B} = \left(\frac{P_A}{P_B}\right)\left(\frac{V_A}{V_B}\right)\left(\frac{T_B}{T_A}\right) = (2)(2)\left(\frac{1}{2}\right) = 2$$

This is the required ratio.

Exercises | Q 13 | Page 73

Answer in brief:

A gas in a cylinder is at pressure P. If the masses of all the molecules are made one-third of their original value and their speeds are doubled, then find the resultant pressure.

SOLUTION

$m_2 = m_{1/3}$, $v_{rms2} = 2v_{rms1}$ as the speeds of all molecules are doubled

Pressure, $P = \frac{1}{3} . \frac{mN}{V} . v^2 rms$

$\therefore P_1 = \frac{1}{3}, \frac{m_1 N}{V} . v^2_{rms\ 1}$ and

$\therefore P_2 = \frac{1}{3}, \frac{m_2 N}{V} . v^2_{rms\ 2}$

$$\therefore \frac{P_2}{P_1} = \left(\frac{m_2}{m_1}\right)\left(\frac{v^2_{rms2}}{v^2_{rms\ 1}}\right)$$

$$= \left(\frac{m_2}{m_1}\right)\left(\frac{v_{rms\ 2}}{v_{rms\ 1}}\right)^2$$

$$= \left(\frac{m_1/3}{m_1}\right)(2)^2 = \frac{4}{3}$$

$$\therefore P_2 = \frac{4}{3}P_1 = \frac{4}{3}P$$

This is the resultant pressure.

Exercises | Q 14 | Page 74

Answer in brief:

Show that rms velocity of an oxygen molecule is √2 times that of a sulfur dioxide molecule at S.T.P.

SOLUTION

$$\frac{M_0(SO_2)}{M_0(O_2)} = \frac{64 kg/mol}{32 kg/mol} = 2$$

The rms speed, $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

$\therefore v_{rms} \propto \frac{1}{\sqrt{M_0}}$ at constant T

$$\therefore \frac{v_{rms}(O_2)}{v_{rms}(SO_2)} = \sqrt{\frac{M_0(SO_2)}{M_0(O_2)}} = \sqrt{2}$$

Thus, $v_{rms}(O_2) = \sqrt{2}\, v_{rms}(SO_2)$

Exercises | Q 15 | Page 74

Answer in brief:

At what temperature will oxygen molecules have same rms speed as helium molecules at S.T.P.? (Molecular masses of oxygen and helium are 32 and 4 respectively).

SOLUTION

Data: T_2 = 273 K, M_{01} (oxygen) = 32×10^{-3} kg/mol,

M_{02} (hydrogen) = 4×10^{-3} kg/mol,

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

The rms speed of oxygen molecules, $v_1 = \sqrt{\frac{3RT_1}{M_{01}}}$ and that of helium molecules,

$$v_2 = \sqrt{\frac{3RT_2}{M_{02}}}$$

when $v_1 = v_2$,

$$\sqrt{\frac{3RT_1}{M_{01}}} = \sqrt{\frac{3RT_2}{M_{02}}}$$

$$\therefore \frac{T_1}{M_{01}} = \frac{T_2}{M_{02}}$$

$\therefore$ Temperature, $T_1 = \frac{M_{01}}{M_{02}} . T_2 = \frac{(32 \times 10^{-3})(273)}{4 \times 10^{-3}}$

= 2184 K

Exercises | Q 16 | Page 74

Answer in brief:

Compare the rms speed of hydrogen molecules at 127ºC with rms speed of oxygen molecules at 27ºC given that molecular masses of hydrogen and oxygen are 2 and 32 respectively.

SOLUTION

Data: M_{01} (hydrogen) = 2 g/mol,

M_{02} (oxygen) = 32 g/mol,

T_1 (hydrogen) = 273 + 127 = 400 K,

T_2 (oxygen) = 273 + 27 = 300 K

The rms speed, $v_{rms} = \sqrt{\frac{3RT}{M_0}}$,

where M_0 denotes the molar mass

$$\therefore \frac{v_{rms1}(hydrogen)}{v_{rms2}(oxygen)} = \sqrt{\left(\frac{T_1}{T_2}\right)\left(\frac{M_{02}}{M_{01}}\right)}$$

$$= \sqrt{\left(\frac{400}{300}\right)\left(\frac{32}{2}\right)} = \sqrt{\left(\frac{4}{3}\right)(16)}$$

$$= \frac{(2)(4)}{\sqrt{3}} = \frac{8}{\sqrt{3}}$$

Exercises | Q 17 | Page 74

Find the kinetic energy of 5 litres of a gas at STP, given the standard pressure is 1.013 × 10^5 N/m^2.

SOLUTION

Data: P = 1.013 × 10^5 N/m^2, V = 5 litres = 5 × 10^{-3} m^3

$$E = \frac{3}{2}PV$$

$$= \frac{3}{2}\left(1.013 \times 10^5 \frac{N}{m^2}\right)\left(5 \times 10^{-3} m^3\right)$$

= 7.5 × 1.013 × 10^2 J

= 7.597 × 10^2 J

This is the required energy.

Exercises | Q 18 | Page 74

Calculate the average molecular kinetic energy (i) per kmol (ii) per kg (iii) per molecule of oxygen at 127°C, given that the molecular weight of oxygen is 32, R is 8.31 J $mol^{-1}K^{-1}$ and Avogadro's number N_A is 6.02 × 10^{23} molecules mol^{-1}.

SOLUTION

Data: T = 273 + 127 = 400 K,

molecular weight = 32 ∴ molar mass = 32 kg/kmol,
R = 8.31 $Jmol^{-1}K^{-1}$,
N_A = 6.02 × 10^{23} molecules mol^{-1}

i) The average molecular kinetic energy per kmol of oxygen = the average kinetic energy per mol of oxygen × 1000

$$= \frac{3}{2}RT \times 1000 = \frac{3}{2}(8.31)(400)(10^3)\frac{J}{kmol}$$

$= (600)(8.31)(10^3) =$ **4.986×10^6 J/kmol**

(ii) The average molecular kinetic energy per kg of oxygen

$$= \frac{3}{2}\frac{RT}{M_0} = \frac{4.986 \times 10^6 J/mol}{32kg/kmol}$$

= 1.558×10^5 J/kg.

(iii) The average molecular kinetic energy per molecule of oxygen

$$= \frac{3}{2}\frac{RT}{N_A} = \frac{4.986 \times 10^6 J/mol}{6.02 \times 10^{23} molecule/mol}$$

= 82.82×10^{-21} J/molecule

Exercises | Q 19 | Page 74

Calculate the energy radiated in one minute by a blackbody of surface area 100 cm^2 when it is maintained at 227°C. (Given: $\sigma = 5.67 \times 10^{-8}$ $W/m^2.K^4$)

SOLUTION

Data: t = one minute = 60 s, A = 100 cm^2

$= 100 \times 10^{-4}$ $m^2 = 10^{-2}$ m^2, T = 273 + 227 = 500 K,
$\sigma = 5.67 \times 10^{-8}$ $W/m^2.K^4$

The energy radiated, $Q = \sigma AT^4t$

$= (5.67 \times 10^{-8})(10^{-2})(500)^4(60)$ J

$= (5.67)(625)(60)(10^{-2})$J = 2126 J

Exercises | Q 20 | Page 74

Energy is emitted from a hole in an electric furnace at the rate of 20 W when the temperature of the furnace is 727°C. What is the area of the hole? (Take Stefan's constant σ to be 5.7×10^{-8} Js^{-1} $m^{-2}K^{-4}$.)

SOLUTION

$$\frac{Q}{t} = 20\text{W},\ T = 273 + 727 = 1000\ K$$

$\sigma = 5.7 \times 10^{-8}\ Js^{-1}\ m^{-2}K^{-4}$.

$$\frac{Q}{t} = \sigma A T^4$$

∴ The area of the hole,

$$A = \frac{Q/t}{\sigma T^4} = \frac{20}{(5.7 \times 10^{-8})(10^3)^4}\ m^2$$

$$= \frac{20 \times 10^{-4}}{5.7} = 3.509 \times 10^{-4}\ m^2$$

Exercises | Q 21 | Page 74

The emissive power of a sphere of area 0.02 m^2 is 0.5 kcal $s^{-1}m^{-2}$. What is the amount of heat radiated by the spherical surface in 20 seconds?

SOLUTION

Data: R = 0.5 kcal $s^{-1}m^{-2}$, A = 0.02 m^2, t = 20 s

Q = RAt = (0.5)(0.02)(20) = 0.2 kcal

This is the required quantity.

Exercises | Q 22 | Page 74

Compare the rates of emission of heat by a blackbody maintained at 727°C and at 227°C, if the black bodies are surrounded by an enclosure (black) at 27°C. What would be the ratio of their rates of loss of heat?

SOLUTION

Data: T_1 = 273 + 727 = 1000 K,

T_2 = 273 + 227 = 500 K,

T_0 = 273 + 27 = 300 K

(i) The rate of emission of heat, $\frac{dQ}{dt} = \sigma AT^4$.

We assume that the surface area A is the same for the two bodies.

$$\therefore \frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4$$

$$= \left(\frac{1000}{500}\right)^4 = 2^4 = 16$$

(ii) The rate of loss of heat, $\frac{dQ'}{dt} = \sigma A\left(T^4 - T_0^4\right)$

$$\therefore \frac{(dQ'/dt)_1}{(dQ'/dt)_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4}$$

$$= \frac{10^{12} - 81 \times 10^8}{625 \times 10^8 - 81 \times 10^8}$$

$$= \frac{(10000 - 81) \times 10^8}{544 \times 10^8}$$

$$= \frac{9919}{544} = 18.23$$

Exercises | Q 23 | Page 74

The earth's mean temperature can be assumed to be 280 K. How will the curve of blackbody radiation look like for this temperature? Find out λ_{max}. In which part of the electromagnetic spectrum, does this value lie? (Take b = 2.897 × 10^{-3} m·K.).

SOLUTION

Data: T = 280 K, Wien's constant b = 2.897 × 10^{-3} m.K

$$\lambda_{max} T = b$$

$$\therefore \lambda_{max} = \frac{b}{T} = \frac{2.897 \times 10^{-3} m.K}{280K}$$

= 1.035 × 10^{-5} m

This value lies in the infrared region of the electromagnetic spectrum.

The nature of the curve of blackbody radiation will be the same, but the maximum will occur at 1.035×10^{-5} m.

Exercises | Q 24 | Page 74

A small blackened solid copper sphere of radius 2.5 cm is placed in an evacuated chamber. The temperature of the chamber is maintained at 100 °C. At what rate must energy be supplied to the copper sphere to maintain its temperature at 110 °C? (Take Stefan's constant σ to be 5.67×10^{-8} J s^{-1} m^{-2} K^{-4}) and treat the sphere as a blackbody.)

SOLUTION

Data: r = 2.5 cm = 2.5×10^{-2}m, T_0 = 273 + 100 = 373 K, T = 273 + 110 = 383 K,

$\sigma = 5.67 \times 10^{-8}$ J s^{-1} $m^{-2}k^{-4}$

The rate at which energy must be supplied =

$$\sigma A\left(T^4 - T_0^4\right) = \sigma 4\pi r^2\left(T^4 - T_0^4\right)$$

$$= (5.67 \times 10^{-8})\ (4)\ (3.140)\ (2.5 \times 10^{-2})^2\ (383^4 - 373^4)$$

$$= (5.67)\ (4)\ (3.142)\ (6.25)\ (3.83^4 - 3.73^4) \times 10^{-4}$$

$$= 0.9624\ \text{W}$$

Exercises | Q 25 | Page 74

Find the temperature of a blackbody if its spectrum has a peak at (a) λ_{max} = 700 nm (visible), (b) λ_{max} = 3 cm (microwave region) (c) λ_{max} = 3 cm (FM radio waves). (Take Wien's constant b = 2.897×10^{-3} m.K).

SOLUTION

Data: (a) λ_{max} = 700 nm = 700×10^{-9} m,

(b) λ_{max} = 3 cm = 3×10^{-2} m,

(c) λ_{max} = 3 cm,

b = 2.897×10^{-3} m.K.

$\lambda_{max} T = b$

(a) T = $\frac{b}{\lambda_{max}} = \frac{2.897 \times 10^{-3} m.K}{700 \times 10^{-9} m}$

= 4138 K This is the required temperature.

(b) T = $\frac{b}{\lambda_{max}} = \frac{2.897 \times 10^{-3} m.K}{3 \times 10^{-2} m}$

= 9.66×10^{-2} K

(c) T = $\frac{b}{\lambda_{max}} = \frac{2.897 \times 10^{-3} m.K}{3m}$

= 9.66×10^{-4} K